

CLASS 2 - EXERCISES

EXERCISE 1

$$A = \begin{pmatrix} 2 & 6 & 3 \\ 1 & -2 & \frac{1}{2} \\ -1 & 1 & -\frac{1}{2} \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

COMPUTE

$$A+B, \quad 2A, \quad AC, \quad AB, \quad BA, \quad A^T, \quad B^T, \quad C^T$$

EXERCISE 2

GIVEN A  $m \times m$  MATRIX  $A$ , CONSIDER THE ALTERNATIVE EXPRESSION OF MATRIX-VECTOR PRODUCT:

$$A\mathbf{v} = v_1 A_1 + v_2 A_2 + \dots + v_m A_m \quad \text{WHERE}$$

$v_1, \dots, v_m$  ARE THE ENTRIES OF  $\mathbf{v}$  AND  $A_1, \dots, A_m$  ARE THE COLUMNS OF  $A$ .

USE THIS EXPRESSION TO SHOW THAT THE TRANSFORMATION

$$\mathcal{A}: \mathbb{R}^m \rightarrow \mathbb{R}^m, \quad \mathbf{v} \mapsto A\mathbf{v}$$

IS LINEAR.

EXERCISE 3



- FIND THE MATRIX CORRESPONDING TO THE LINEAR TRANSFORMATION

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x-y \\ x+y \end{pmatrix}$$

IN THE STANDARD BASES OF  $\mathbb{R}^2$ .

- COMPUTE THE KERNEL, RANGE AND RANK OF  $T$ .